Set Theory

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[A concept is] the basic unit of thinking. It can be said that we have a concept of A (or of A-ness) when we are able to distinguish A from whatever is not-A

Giovanni Sartori [1]

QCA is a set-theoretic method. But what are *sets* and how does a *set-theoretic* approach differ from other approaches? At the most basic level, a set can be defined as a group of elements that share certain characteristics. For George Lakoff and Rafael Núñez, one way to see sets is as a "containerlike entity" (Lakoff and Núñez 2000, 140) and James Mahoney describes sets as "boundaries that define zones of inclusion and exclusion" (Mahoney 2010, 7). Given these understandings, it is apparent that sets and concepts are closely interlinked. In fact, most social science theories are grounded in set theory, either explicitly by using the language of necessary and sufficient conditions, or implicitly by invoking the set-theoretic logic without expressly referring to necessity and sufficiency (Goertz and Mahoney 2012; Schneider and Wagemann 2012).[2]

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1 Sartori (2009: 135).

2 For an inventory of 150 necessary condition hypotheses, see Goertz (2003b).

Sets exist all across the social world. Social kinds of sets are concepts that we use to describe groups of objects with specific characteristics. For instance, "democratic states" or "developed countries". The referent concepts of "democracy" and "development" are not objectively given but created in the human mind. As social science concepts, they may be "essentially contested" (Gallie 1956) and their meaning may change over time. For example, "democracy" means something else today than 100 years ago, when it was not seen as an essential property of democracy to have universal voting rights. Likewise, a country's "development" was long understood solely in terms of its economic strength, whereas measures like the UN's Human Development Index rest on a broader conception that includes education, standard of living, and life expectancy (UN 2018).

What distinguishes a set-theoretic approach from other approaches? To illustrate the difference, we can juxtapose set theory with statistics or a "variable-oriented" approach, to borrow Ragin's term (1987, 53). Whereas the latter usually refers to variables as nouns – as in "height" or "partisanship" – the set-theoretic approach uses nouns and adjectives. Accordingly, it may refer to "tall people" and "left partisanship". Because set-theoretic measures are "calibrated", which means that they are oriented towards a predefined point of reference, there is *more information* entailed in calibrated measures (Ragin 2008). Once we know which concept a given set refers to, we also know how to interpret a case's membership score in that set. There are also differences in the way the data is used. Set-theoretic approaches refer to set membership scores, bounded between 0 and 1, whereas variable-oriented approaches typically work with unbounded numbers. To arrive at set membership scores, a researcher's raw data first needs to be *calibrated* according to specified criteria (we will look into calibration in detail in Chapter 4). These approaches thus either emphasize qualitative *differences in kind* or quantitative *differences in degree*. Finally, whereas operations on sets follow the logical rules of Boolean algebra and set theory, variables are manipulated following standard mathematical rules.[3]

As noted in the introduction, QCA is grounded in the works of George Boole, a British 19th century mathematician and logician whose *Laws of Thought* (Boole 1854) established the foundation for what was later termed Boolean algebra. Boole's

[3] This difference should not be overdrawn, as set theory can be seen as a "unifying theory for mathematics" (Cunningham 2016, : ix).

approach uses variables that occur in only two states: "true" (present) or "false" (absent). This conception proved central to the development of electronic switching circuits and Boolean algebra was soon widely used across the applied sciences (Whitesitt 2010), and it also made inroads into the social sciences. Charles Ragin adopted these principles to develop a "Boolean approach" for comparative studies (Ragin et al. 1984), which then became known as "Qualitative Comparative Analysis" (Ragin 1987). What is important for the aims of qualitative comparison is that Boolean algebra allowed for set-theoretic operations, the construction of truth tables, and their minimization to derive solution terms (see Chapter 6).

This chapter lays out the essentials of set-theoretic approaches. Starting with the distinction between crisp sets and fuzzy sets, it introduces Boolean algebra, set-theoretic operations, formal notation, and the ideas behind truth tables. This is followed by a discussion of the concepts of necessary and sufficient conditions, which form the nuts and bolts of QCA.

Crisp Sets and Fuzzy Sets

The Boolean use of binary categories meant that QCA was, in its original form, limited to working with "crisp sets", where 1 indicated the presence of a condition and 0 indicated its absence. This emphasized *qualitative* differences. Either a case belonged to a given set, or it did not. It is apparent that this procedure entails a loss of nuance, as any information needs to be coded as either "1" or "0", and no further distinctions can be made (Mello 2014; Rihoux and De Meur 2009).

The drawback of crisp-set QCA was overcome with the introduction of "fuzzy sets" (Ragin 2000), which allowed for graded set membership, as any scores between 0 and 1 became possible. Fuzzy logic was developed by Lotfi Zadeh (1965) as an extension of traditional set theory to tackle the problem of complex and imprecise concepts. Zadeh's work sparked a revolution in computer technology (McNeill and Freiberger 1993) and fuzzy sets have also made their way into the social sciences (Smithson and Verkuilen 2006), including linguistics (Lakoff 1973), and many other areas.

Fuzzy-set QCA combines qualitative and quantitative dimensions. Based on substantive and theoretical knowledge of their topic, researchers establish three "qualitative anchors" that determine whether a case is "fully in" a given set (resulting in a fuzzy value of 1.0), whether it is "fully out" a given set (fuzzy value of 0.0), or whether it is "neither in nor out" a given set (fuzzy value of 0.5). The latter is also known as the "point of maximum ambiguity", which means that based on the available evidence it is not possible to say whether the case is rather inside or outside a respective set (Ragin 2000; 2008).

Fuzzy sets clearly were a major step forward in the evolution of QCA. They allowed for fine-grained differentiation and, addressing concerns of some of the method's critics, showed that QCA did not have to be rooted in a deterministic understanding of causality. They also paved the way for "measures of fit" that enabled researchers to assess the quality and robustness of their analytical results. Finally, they enabled the transformation of quantitative data into fuzzy sets, through software-based methods of calibration.

Irrespective of these advantages of fuzzy sets, it is important to note that the Boolean logic still applies – cases are treated as either "inside" or "outside" of a given set and the differences in degree only indicate that they are *more or less* in or out of those sets. The researcher still has to determine the central criterion how to distinguish cases that are rather inside a set from those that are rather outside.

This challenge has been described as "Sorites Paradox" or paradox of the heap (derived from the Greek word *sorós*). For this we assume that 1,000,000 grains of sand form a heap. Now removing a single grain of sand will not change this, as 999,999 grains of sand are also a heap, and so it will be if another grain of sand is removed. This prompts the question at which point the heap will turn into a "non-heap" – how many grains do we have to remove until the heap disappears? To solve the paradox, we need to determine strong criteria to distinguish one from the other, which is challenging for the heap of sand, because any numerical threshold may appear arbitrary. For social science concepts it can actually be helpful, because it lets us think hard about "difference-makers" and the ontological characteristics of concepts.

A common misconception about fuzzy sets – which remains surprisingly prevalent – is that these somehow reflect probabilities. Clearly, that is not the case. Let us take an example to illustrate the difference (see Figure 1). Suppose you have two red chilis of identical appearance. You have no other information but that the first chili has a 1% chance of being a very spicy chili, whereas the second chili has a set-theoretic membership score of 0.01 in the set "very spicy chilis". You want to cook a non-spicy dish with mild chili as an ingredient. Which of these chilis would you choose?

At first glance, it may appear that there is no difference between the two options. However, on closer examination it should become clear that for chili A the odds are 1 out of 100 that you will pick a very spicy chili. For chili B the information is not a probability – you *know for your sure* that chili B is almost entirely outside the set “very spicy chilis” – so this would be the safer choice in the given scenario. Fuzzy sets thus attribute a discrete score to a specific case and there is no uncertainty or probability involved.

Figure 1: Probabilities and Fuzzy Sets

Chili A	Chili B
	
1% chance of being a very spicy chili	0.01 membership in the fuzzy set "very spicy chilis"

Set-Theoretic Operations

Boolean algebra entails three set-theoretic operations.[4] The logical operator AND refers to the intersection or, to use the term from propositional logic, “conjunction”, between two sets. Logical OR refers to the union or “disjunction” between the respective sets, whereas logical NOT describes the negation or “complement” of a set.

What can be confusing about QCA is that it draws on several different notational systems. Depending on a subfield’s custom, it may be preferred to use symbols and terms from Boolean algebra, the logic of propositions, or formal set theory. Admittedly, the existence of several different terms for the same set-theoretic operations complicates things. In this book, an effort is made to keep formal notation to a minimum and to consistently use the same notation throughout. As a “translation guide”, Table 1 summarizes the terms and notational forms that are commonly used,

[4] For comprehensive introductions to the mathematical foundations of Boolean algebra and set theory, see Whitesitt (2010) and Cunningham (2016).

depending on whether the reference point is Boolean algebra, set theory, or propositional logic.[5]

Table 1: Logical Operators and Notational Systems

Logical Operator	*Boolean Algebra*	*Set Theory*	*Propositional Logic*
AND	Multiplication A * B	Intersection $A \cap B$	Conjunction $A \wedge B$
OR	Addition A + B	Union $A \cup B$	Disjunction $A \vee B$
NOT	Negation 1 – A	Complement ~ A	Negation $\neg A$

Boolean operations are best illustrated with an example. Table 2 summarizes hypothetical data for three cases, two crisp sets (A and B), two fuzzy sets (C and D), and the Boolean operators AND, OR, and NOT.

Table 2: Illustration of Boolean Operations with Crisp and Fuzzy Sets

	Crisp Sets		*Fuzzy Sets*		*Boolean AND*		*Boolean OR*		*Boolean NOT*	
	A	B	C	D	A * B	C * D	A + B	C + D	~ A	~ C
Case 1	1	0	0.9	0.3	0	0.3	1	0.9	0	0.1
Case 2	1	1	0.7	0.8	1	0.7	1	0.8	0	0.3
Case 3	0	0	0.2	0.4	0	0.2	0	0.4	1	0.8

[5] For summaries of notational systems see also Cunningham (2016, 6); Schneider and Wagemann (2012, 54); Smithson and Verkuilen (2006, 6).

Let us assume we are interested in cases that share two characteristics. Only if both of these are present do we expect to see our outcome (or "dependent variable"). In set-theoretic terms this is expressed as: A * B, where "*" stands for "AND" (not to be confused with the multiplication sign from mathematical algebra). A case's set membership in the expression A * B is the *minimum* value across the two sets. So Case 1 and Case 3 both receive a value of "0" in the expression A * B, because their "weakest link" or lowest common denominator has that value (see Table 2).

Alternatively, we may be interested in a situation where cases have at least one out of two characteristics – either of which we expect to lead towards our outcome. This is expressed as: A + B, where "+" stands for "OR". Again, this should not be confused with the mathematical operator for addition. Accordingly, a case's set membership in the expression A + B is the *maximum* value across the two sets. It follows that Case 1 and Case 2 get the same value for A + B, because each has at least one set with a value of 1.

The final set-theoretic operator refers to the negation of set. Once you know that a case holds membership in a given set you can also describe its membership in the "non-set", or the negation of that set. It is 1 – A. Here, the minus sign actually refers to the mathematical operator of subtraction. The last two columns of Table 2 show the values for "non-A" and "non-C". Note that the values for a condition and its negation must always add up to 1.

For crisp sets, Venn diagrams are the most commonly used approach to visualize set-theoretic relationships (Goertz and Mahoney 2012; Schneider and Wagemann 2012). The following figures depict the previously discussed operations. Figure 2 shows the logical intersection "AND" of the sets A and B. Figure 3 shows the logical union "OR" of the same sets. Note that this set-theoretic operator is based on an inclusive conception of "OR".[6]

[6] One could also calculate the *set difference* between sets A and B, which is expressed as "A \ B" or "A minus B", but this is rarely needed in a QCA context (cf. Cunningham 2016, 4).

Figure 2 Logical Intersection of Set A and B (A * B, A ∩ B, Logical AND)

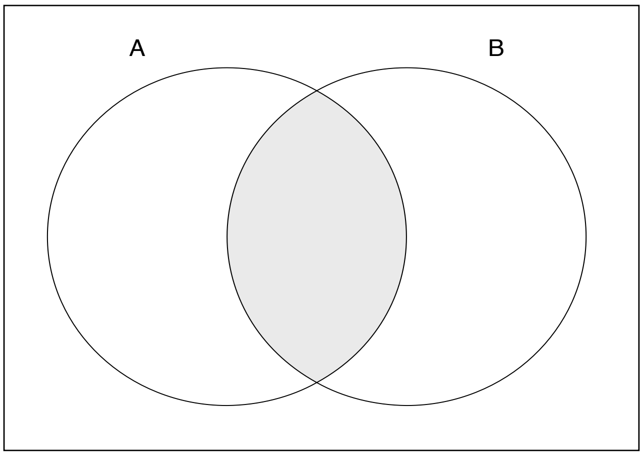
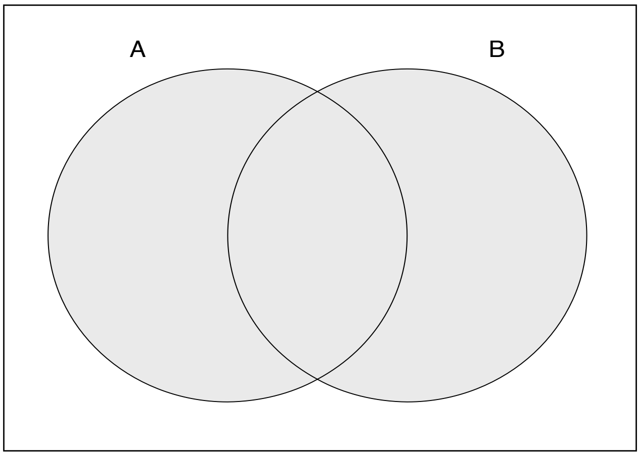


Figure 3: Logical Union of Set A and B (A + B, A ∪ B, Logical OR)



Truth Tables

A central analytical tool for QCA is the *truth table*. The analytical procedure will be discussed in detail in Chapter 6, but for now it suffices to introduce the key features of truth tables and to describe how they differ from ordinary data matrices.

The truth table shows the number of logically possible combinations for a given study. Each row in the truth table refers to a specific combination of conditions. The number of rows of the truth table equals the total number of possible combinations of conditions. The formula for this is 2^k, where k is the number of conditions that are included in a given study. This means that for two conditions there will be eight rows, three conditions will result in 16 rows, and so forth. Individual cases are assigned to the row that reflects their scores for the respective conditions (Mello 2017).

Table 5 shows a simple example with only two conditions, an outcome, and 14 hypothetical cases that distribute across the four rows of the truth table. We can see from the table that all four rows are populated with cases. That is not always so. With more conditions, truth tables become exponentially larger and the likelihood increases that the empirical cases will not fill all of the logically possible combinations. That is the issue of *limited diversity*, which we will return to in later chapters.

We can also see that the combinations represented by the top two rows are associated with a positive outcome, whereas the lower two rows are associated with a negative outcome. Based on the empirical cases, we can say that A * B (Row 1) and A * ~B (Row 2) lead to the outcome Y. This kind of information forms the basis for the *truth table analysis*, where a minimization algorithm will be applied (via software) to reduce the Boolean expressions and gain more parsimonious solution terms for the explanation of an outcome (see Chapter 6).

Table 5: A Simple Truth Table

Conditions		Outcome	Number of Cases
A	*B*	*Y*	
1	1	1	5
1	0	1	3
0	1	0	2
0	0	0	4

Necessary and Sufficient Conditions

As a set-theoretic method, the analytical procedure of QCA aims at the identification of necessary and sufficient conditions.[7] For Goertz and Mahoney (2012: 12), the logic of necessity and sufficiency is what characterizes the heart of *qualitative* research.

[7] While QCA can be used for different purposes (Berg-Schlosser et al. 2009, 15), most publications conduct some form of theory-guided analysis aimed at the identification of necessary and sufficient conditions (Mello 2013), which resonates with QCA's "orientation towards causal inference" (Wagemann 2017, 11).

A *necessary condition* means that a factor is always present when the outcome of interest occurs. For example, you may find that countries that experienced democratic breakdown always did so under conditions of economic crisis. This does not mean that an economic crisis causes a democratic breakdown in itself (there may be other factors contributing to that outcome), but whenever there has been a breakdown of democracy it was preceded by an economic crisis. In formal terms, the condition "economic crisis" would thus be a *superset* of the outcome "democratic breakdown".

A *sufficient condition* means that whenever a factor is present, the outcome is also present. For instance, a government change may be a sufficient condition for policy change. This does not mean that policy change can only occur after a government change (as there will be a variety of other factors that also lead to policy change), but for a given population of cases it may be that whenever a new government has come to power, there has also been a policy change. In formal terms, the condition "government change" is thus a *subset* of the outcome "policy change".

In formal notation, a necessary condition is indicated by a left-pointing arrow, directed from the outcome towards the condition that is necessary: $A \leftarrow Y$, whereas a sufficient condition is indicated by a right-pointing arrow, directed from the condition that is sufficient towards the outcome: $B \rightarrow Y$.

While the connection is often not made explicit, many social science theories are based on an understanding of necessary and sufficient causation (Goertz 2003a; Goertz and Starr 2003). Here are several examples from studies in international relations, political science, and economic history:

(1) "If not a necessary condition, nuclear deterrence may be interpreted as a *sufficient condition* for peace" (Gleditsch 1995, 543, original emphasis).

(2) "The introduction of universal suffrage *led almost everywhere* (The United States excepted) to the development of Socialist parties" (Duverger 1954, 66, emphasis added).

(3) "One cannot have the productivity of an industrial society with political anarchy. But while such a state is *a necessary condition* for realizing the gains from trade, it obviously is *not sufficient.*" (North 1984, 259, emphasis added).

(4) "Thus, international pressure was a *necessary condition* for these policy shifts. On the other hand, *without domestic resonance*, international forces *would not have sufficed* to produce the accord, no matter how balanced and intellectually persuasive the overall package." (Putnam 1988, 430, emphasis added).

(5) "In the rationalist perspective, however, a community of basic political values and norms is *at best a necessary condition* of [EU] enlargement [...] By contrast, in the sociological perspective, sharing a community of values and norms with outside states is *both necessary and sufficient* for their admission to the organization." (Schimmelfennig 2001, 61, emphasis added).

What these five quotes illustrate, and many more examples could be given, is that authors frequently base their arguments on a set-theoretic logic. The first quote from Nils Petter Gleditsch about nuclear deterrence is straight-forward in its mention of a condition that is sufficient but not necessary. The second quote from Maurice Duverger about electoral rules differs from the first because it does not use the language of necessary and sufficient conditions. However, its essence is set-theoretic: universal suffrage is seen as an (almost) sufficient condition for the development of Socialist parties. The third quote from Douglass North turns the perspective by highlighting state institutions as a necessary condition for economic productivity. North also points out that while this condition is necessary, it is not sufficient to bring about the outcome.

Similarly, Robert Putnam's quote focuses on international pressure as a necessary condition for policy shifts. Interestingly, Putnam also argues in combinatorial ways, emphasizing that it needed international pressure *and* domestic resonance, which together form a sufficient condition for policy shifts. Finally, the quote from Frank Schimmelfennig describes different expectations concerning necessity and sufficiency, based on competing theoretical perspectives. While rationalist theory expects a weak necessary condition, social constructivism expects a condition that is necessary and sufficient.

Assessing Necessity and Sufficiency

An easy and intuitive way to explore set-theoretic relationships for binary or crisp-set data are two-by-two tables (Goertz and Mahoney 2012; Schneider and Wagemann 2012). Looking at the distribution of empirical cases across the four cells of a 2x2 table allows one to see whether there may be a set-theoretic relationship of necessity and/or sufficiency.

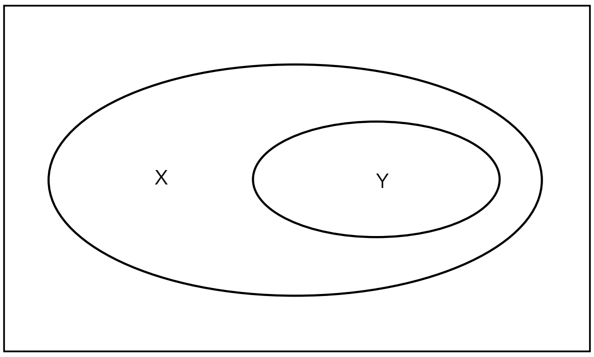
Table 3 shows the cells that are relevant if one expects a necessary condition. The focus is on the rightwards column where the outcome is present [1], because this is what we are interested in. If there are many cases that show the outcome and the condition, and no cases that show the outcome but do not show the condition, then this suggests that we have identified a necessary condition. Note that cases without the outcome are not important for the analysis of necessary conditions, regardless of how many there are.

Table 3: 2x2 Table and Relevant Cells for a Necessary Condition

		Outcome 0	Outcome 1
Condition	1	Not important	Cases
	0	Not important	No cases

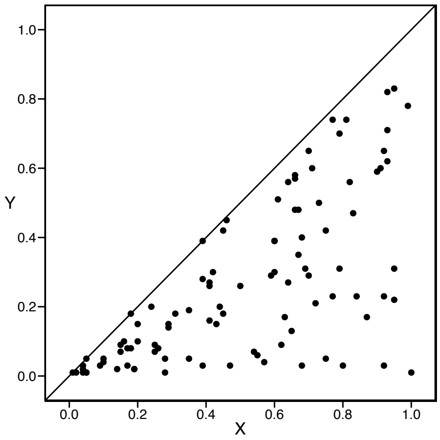
The set-theoretic relationship of necessity, based on crisp-set data, can also be visualized with a Venn diagram, shown in Figure 4. Because a necessary condition is a *superset* of the outcome, the circle for the condition X fully encloses the outcome set Y. Every case that holds membership in Y also holds membership in X. However, the reverse is not true, as can be seen from the circles, where many cases hold membership in X but not in Y.

Figure 4: Venn Diagram for a Necessary Condition (Crisp Sets)



What about fuzzy sets? The best way to visualize these are XY plots, with the condition on the x-axis and the outcome on the y-axis. Figure 5 shows a perfect necessary condition, as all the cases (shown in black dots) hold values for the outcome Y that are lower or equal to their respective values for the condition X, mirroring the expected subset relationship. This is reflected in the diagonal line. This line separates cases with values that are equal to or higher for the outcome than for the condition ($Y \geq X$) from those that are equal to or lower for the outcome than for the condition ($Y \leq X$). Cases with equal values for the condition and the outcome are situated exactly on the diagonal line.

Figure 5: XY Plot for a Necessary Condition (Fuzzy Sets)



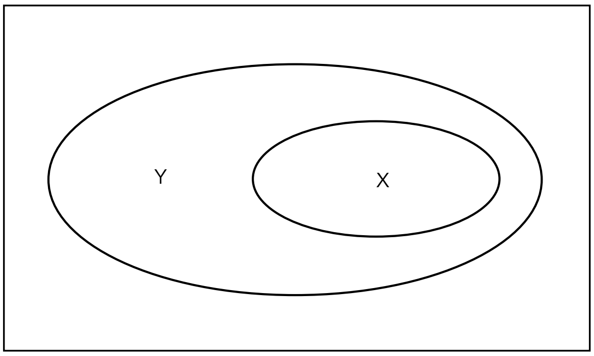
For sufficient conditions, the procedure is similar, but inverted. Table 4 shows the cells that are relevant if one expects a sufficient condition. Now the focus rests on the upper row where the condition is present [1], since this is what we want to assess. If there are many cases that show the condition and the outcome, and no cases that show the condition but do not show the outcome, then this suggest that we have identified a sufficient condition. Naturally, cases without the condition are not important for the analysis of sufficiency.

Table 4: 2x2 Table and Relevant Cells for a Sufficient Condition

		Outcome	
		0	1
Condition	1	No cases	Cases
	0	Not important	Not important

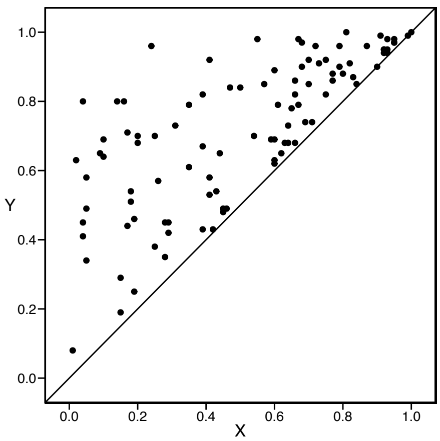
Again, the set-theoretic relationship of sufficiency for crisp-set data can also be visualized with a Venn diagram, shown in Figure 6. Because a sufficient condition is a *subset* of the outcome, the circle for the condition X is fully enclosed by the outcome set Y. Every case that holds membership in X also holds membership in Y. Yet, this does not account for all instances of Y, because there is an area that is covered by X (but which may be accounted for by other conditions).

Figure 6: Venn Diagram for a Sufficient Condition (Crisp Sets)



As for fuzzy sets, Figure 7 shows a perfect sufficient condition, as all the cases hold values for the outcome Y that are higher or equal to their respective values for the condition X, in line with the expected set-theoretic relationship.

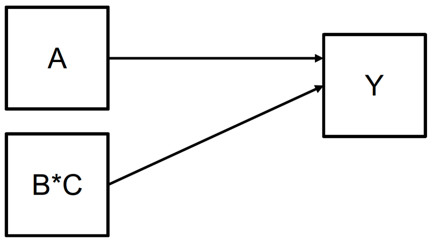
Figure 7: XY-Plot for a Sufficient Condition (Fuzzy Sets)



Causal Complexity

Necessary and sufficient conditions naturally lead to a discussion of "causal complexity", which is one of the particular strengths of the QCA approach (Ragin 2008, 124; Schneider and Wagemann 2012, 78). But what is meant by the term causal complexity? As highlighted in the introductory chapter, QCA rests on three methodological assumptions: *conjunctural causation*, *equifinality*, and *causal asymmetry*. Together, these assumptions comprise causal complexity. Ragin offers the following definition: "Causal complexity is defined a situation in which a given outcome may follow from several different causal 'recipes'." (Ragin 2008, 124). These "recipes" or "paths" may comprise different combinations of factors. As the simplest example, suppose you have three conditions A, B, and C and an outcome Y. Your analysis may reveal that condition A is *individually sufficient* for the outcome, whereas conditions B and C are *jointly sufficient* (as shown in Figure 8). This means that you have a situation of equifinality (there is more than one path to the outcome) and conjunctural causation (two conditions combine to bring about the outcome).

Figure 8: Causal Complexity



In formal terms, the conditions B and C are "INUS" conditions, meaning they are "an *insufficient* but *necessary* part of a condition, which is itself *unnecessary* but *sufficient* for the result" (Mackie 1965, 245). INUS conditions are the nuts and bolts of QCA solutions (addressed in later chapters), as you typically find combinations of two or more conditions that together are sufficient for the outcome.

The complement to INUS conditions are "SUIN" conditions, which are "a *sufficient* but *unnecessary* part of a factor that is *insufficient* but *necessary* for an outcome" (Mahoney et al. 2009, 126). SUIN conditions can be conceived of as "equivalent indicators" (Mello 2017, 127) that form a necessary condition. For example, the condition A may be necessary for the outcome and there are two conditions F and G, each of which can bring about A. In this scenario, F and G would each be SUIN conditions. We will return to INUS and SUIN conditions in later chapters, when discussing concrete examples.

Summary

This chapter introduced key elements of set theory, as the foundation for QCA. Starting with a discussion of what characterizes set-theoretic approaches, the chapter moved to crisp and fuzzy sets, Boolean algebra and set-theoretic operations, and truth tables as a key analytical device in QCA. The chapter closed with a presentation of necessary and sufficient conditions and causal complexity, their usage in social science theory, and ways to assess empirical data for patterns of necessity and/or sufficiency, and to visualize set-theoretic relationship for crisp and fuzzy set data.

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